# Abstraction reflective student in problem solving of Mathematics based cognitive style

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**Abstract**
The student's reflective abstraction ability in solving problems is necessary because the result of a person's reflective abstraction is a scheme used to understand something, finding solutions or solving problems. Besides, reflective abstractions are essential to higher mathematical, logical thinking as they occur in logical thinking in children. Therefore, to develop a reflective abstraction notion of high-level mathematical thinking, it is necessary to separate what is an essential feature of reflective abstraction, reflect its rules on higher mathematics, recognize and reconstruct it so that a similar theory of knowledge Mathematics and its instructions. While research that will researchers do is to know how the process of reflective abstraction of students in solving problems in terms of cognitive style. This is because the cognitive style is closely related to how to receive and process all information, especially in learning. The various trends in their learning can be identified and then classified whether the child belongs to an independent field cognitive style (thinking tends to have the independence of views) or field dependent.

**Keywords:**
Reflective abstraction; cognitive style; field-dependent; field-independent.

## 1 INTRODUCTION

In thinking for the formation of the concept of one by using abstraction. Subanji (2015) says that abstraction occurs when from some objects than 'aborted' the characteristics or properties of the object that are considered unimportant, and ultimately only noticed or taken the important properties shared. Abstraction begins with a set of objects; then the object is grouped by important properties and relationships, then aborted nature and relationships that are not important. Paschos (2006) explains that abstraction is a vertical reorganization activity of mathematical concepts that have been constructed previously through a new mathematical structure. New mathematical objects are constructed through the establishment of such relationships to find new generalizations, evidence, or strategies on problem-solving.

Piaget (in Gray & Tall, 2007; Ozmantar & Monaghan, 2007) distinguishes three forms of abstraction, namely empirical abstraction, psycho-empirical abstraction, and reflective abstraction. Empirical abstraction is the process of acquiring knowledge of the properties of the object. The process relates to a subject's experience when viewing objects through direct experience by looking at the visible properties of an object. However, the knowledge that is formed is internal within the subject. The process of pseudo empirical abstraction occurs when the subject is confronted with an object then finds the properties of the object through the process of imagining an action imposed on the object.

The subject tries to configure the object in the space as well as to examine possible relationships. The third abstraction is a reflective abstraction also called Piaget as the general coordination of the actions that originate in the subject and the whole is internal. This process leads the subject to a different kind of generalization constructive and produces a new form of synthesis between specific rules in obtaining new insights. Student abstraction abilities in solving problems are necessary, following Goodson-Espy's opinion (2015) which says that the results of a person's mental abstraction are schemes used to understand things, find solutions or solve problems. In problem-solving activities in a situation, students often link the activity to the next problem-solving situation. Piaget (Tall, 2013) believes that reflective abstraction is essential for higher mathematical logic thinking as occurs in logical thinking in children. Therefore, to develop a reflective abstraction notion of high-level mathematical thinking, it is necessary to separate what constitutes an essential feature of reflective abstraction, reflect its rules on higher mathematics, recognize and reconstruct it so that a similar theory of knowledge mathematics and its instructions.

The reflective abstraction in this study is to emphasize Piaget's opinion that there is a general coordination of action that describes the construction of an individual's mathematical logic structure in constructing a new concept. The construction of a new concept in this research is a reinforcement of the concept that is on solving math problems. Piaget in Paschos and Farmaky (2006) distinguish a reflective abstraction in four types of construction processes: interiorization, as an internal construction process, i.e. how to make sense of perceived phenomena. Coordination or composition of two or more new construction processes. The encapsulation or conversion of a process (dynamic) into an object (static), in a sense, that, " action or operation becomes the object that is generated from thought or assimilation" Piaget considers that " Math entities move from one level to another, an operation like 'entity' will be the object of theory.". When the subject learns to apply the existing scheme to a broader collection of phenomena, then we say that the scheme is Generalization. Generalization can also occur when a process is formulated into an object. The scheme will remain the same unless it has a broader application. Piaget calls all this as reproductive or generalization assimilation and is called extensional generalization.

Cognitive style is one of the characters of students that is very important and influential, especially on the achievement of their learning achievement. Cognitive style is concerned with how they learn through their inherent ways and become distinctive to everyone. Cognitive style is closely related to how to receive and process all information, especially in learning. The various trends in their study can be identified and then classified whether the child belongs to an independent field cognitive style (thinking tends to have the independence of views) or filed dependent. Based on the analysis of student duties on the initial study of 32 students at the Islamic University of Malang turned out to have different results of students who are cognitive field-independent and independent cognitive-style students. There is a difference in completing the task based on the construction of its reflective abstraction. There is a difference in the process of coordination, student- style cognitive field-dependent with students who are a cognitive-style independent field. The reflective abstraction has been studied, among others, by Farmaki and Paschos (2006), Zollman, Cappetta, and Robert (2013), Sudirman (2014), Sopaemena (2016). These studies only examine the emergence of types of reflective abstraction construction processes that almost all begin by interiorization and end with generalizations. This research describes the process of student abstraction in solving math problems in terms of cognitive style.

## 2 Theoretical Framework: Reflective Abstraction and Cognitive Style

Piaget presents his ideas on the three abstract forms reviewed by (Gray &Tall, 2012; Ozmantar & Monaghan, 2007). According to Piaget, there are three forms of abstraction, namely empirical abstraction, psycho-empirical abstraction, and reflection abstraction. Empirical abstraction is the process of acquiring knowledge of the properties of various kinds of objects. The process relates to a subject's experience when viewing objects through direct experience by looking at the visible properties of an object. However, the knowledge that is formed is internal within the subject. According to Piaget, this type of empirical abstraction can usher in the ability to extract the general properties of objects and deliver on the continued generalization.

The process of pseudo empirical abstraction occurs when the subject is confronted with an object then finds the properties of the object through the process of imagining an action imposed on the object. The subject tries to configure the object in the space as well as to examine possible relationships. In the end, the reflective abstraction also called Piaget as the general coordination of the actions that are subject to the subject itself, and the whole is internal. This process leads the subject to a different kind of generalization constructive and produces a new form of synthesis between specific rules in obtaining new insights. The result of reflective abstraction is the schema (mental structure) of knowledge at each stage of development and the reflective abstraction fuses the schema of the corresponding action pattern.

The difference between empirical abstraction and theoretical abstraction in the learning process, especially fractional numbers can be seen in the following example. For example, in studying the concept of fractions, based on the theory of empirical abstraction, the process is the child recognizing various forms of fractional representation models first, such as the fractional model of the 'whole,' the 'part of a set' model. In this process the child recognizes the same characteristics of experiences with real objects (enactive representations of Bruner's stages), although the same characteristics are only roughly linear and are produced from various contexts, but from which the concepts will be known. While theoretical abstraction, students are introduced beforehand about the concept of dividing the same (fair sharing) and the concept of the fraction with the conventional symbol, which is "a / b" (symbolic representation at Bruner stage).

Based on the notion of reflective abstraction, an indication of the process of reflective abstraction in learning mathematics can be observed from the following activities: (1) Identify the characteristics of the object through direct experience. (2) Identify the characteristics of objects that are manipulated or imagined. (3) Make a generalization. (4) Represents mathematical ideas in language and mathematical symbols. (5) Releasing the material properties of an object or idealizing. (6) Making connections between processes or concepts to form a new understanding. (7) Applying the concept to the appropriate context. (8) Manipulate abstract mathematical objects. Every individual is psychologically different about how to process information and organize its activities. These differences affect the quantity and quality of the results of activities undertaken included in student learning activities. This difference is called the cognitive style (cognitive style). The cognitive style refers to the way people obtain information and use strategies to respond to surrounding environmental stimuli.

According to Angeli (2013), cognitive style is a different way to see, recognize, and organize information. Everyone has a particularly preferred way of processing and organizing information in response to its environmental stimuli. Even further Angeli explains everyone could respond quickly and some are slow. These ways of responding also relate to personal attitudes and qualities. A person's cognitive style can show individual variations in attention, receiving information, remembering, and thinking that arise or differ between cognition and personality. Cognitive styles are patterns formed by the way they process information, tend to be stable, though not necessarily unchangeable. Meanwhile Riding and Rayner (2012) describe a cognitive style is an approach that the individual loves consistently in organizing and describing information. Mortomoore (2008) argues that cognitive style is an individual habit in processing information. The same thing is also disclosed Allport (2010) cognitive style is the habit or way that individuals prefer to process information.

The above explanation shows that the cognitive style is a psychological dimension as a person's character in responding to all the information it receives. So, it can be understood that cognitive style is the way that individuals prefer consistently in obtaining, organizing, describing, and processing information. The cognitive style itself can be divided into two, namely the first based on differences in psychological aspects consisting of field-dependent and independent field, the second is based on the time of concept understanding consisting of impulsive and reflective style. But in this research used is cognitive style field-independent and field-dependent.

## 3 METHOD

This research is qualitative. This research will explore understanding a cognitive-style student of FI and FD on mathematical concepts. In this paper, the concept which will be constructed is the concept of ratio. Students are given a related matter sketch chart function. Based on his work, then conducted interviews. The interview depth here aims to uncover an understanding of the concept of the ratio. The subject of this research is the students of the Mathematics Education Study Program of Islamic Malang University who have taken Calculus I course. Students are given an Embedded Group test Figures Test (GEFT) to determine its cognitive style. The results of this test are used to determine the subject of research belonging to an independent field (FI) or style cognitive field-dependent (FD). Based on the results of the GEFT test, this study selected one person from the FI group and one from the FD group.

The main instrument in this study is the researchers themselves. Therefore, now data collection in the field, researchers participate during the research process and follow actively the activities of research subjects related to data collection through interviews. The main instrument in this study is the researchers themselves. Therefore, now data collection in the field, researchers participate during the research process and actively follow the activities of research subjects related to data collection through interviews. Data collection is done by giving a problem in the Task Sheet Student to student-related to ratio. From the work that students as the data to be the basis of the implementation of the interview. To obtain a picture of conceptual understanding, carried out the following steps: (1) students given the task to solve the problem, (2) researchers check the results of student work, (3) researchers provide questions related to written answers provided by students through interviews. The results of written and verbal answers (obtained during the interview) later examined its determination or its consistency. If there is inconsistent data, it can be done in the interview again. The data obtained during the interview was recorded using MP3. Parts of written work and unrecognized tapes, researchers can discuss with the subject after completing all tasks.

## 4 RESULT AND DISCUSSION

This section presents a summary of the interviews and written work of the FI and FD subjects. Due to the limitations of the space provided for this paper, then presented here is a small snippet of long exposure data from the research results.

### 4.1 FI Student (Subject 1)

4.1.1 Interioritation

- Students are able to understand the given problem by being able to write down what is known, which is asked
- Students can determine the given problem in the form of a reversed ratio
- Students use variables to solve problems with x for days, k for chairs and l for cabinet

4.1.2 Coordination

- Students calculate the time for 10 craftsmen by converting them into 1 person by dividing 5
- Students can coordinate between variables x, k, and l in determining the time required to create 30 seats and 36 cabinets

4.1.3 Encapsulation

- Students can set the time required by dividing the same number x (20 k + 24 l) = 3/2 (30 k + 36 l)
- Students can set the x value sought

4.1.4 Generalization

- Students use the x value sought to determine the time required by 10 people to create 30 seats and 36 cabinets ie for 2 ¼ days

### 4.2 FD Student (S2 Subject).

4.2.1 Interiorization

- Students are able to understand the given problem by being able to write down what is known, which is asked
- Students have not been able to determine the given problem in the form
- Students use variables to solve problems with x for days, k for chairs and l for cabinets

4.2.2 Coordination

- The student calculates the time for 10 craftsmen by converting first by multiplying 2/3
- Students can coordinate between variables x, k, and l in determining the time required to create 30 seats and 36 cabinets

4.2.3 Encapsulation

- The student sets the time required by matching the resulting product
- Students are still not precisely determine the calculation of the resulting product

4.2.4 Generalization

- Students use the x value sought to determine the time required by 10 people to make 30 seats and 36 cabinets for 6 days

## 5 CONCLUSION

From the research result of the student's reflective abstraction process in solving the cognitive mathematical problems of FI and FD on the concept of comparison, the following conclusions are obtained:

### 5.1 Students who cognitive FI style (subject S1)

At the stage of interiorization is able to read things related to known problems and be able to identify previous activities related to the given problem, for the subject coordination process F1 able to coordinate the results of interiorization and able to change the information into a mathematical model, for encapsulation able to build the mathematical structure problems in the formation of coordination a and the process of generalization able to use the scheme that formed into objects

### 5.2 Students with cognitive FD style (subject S2)

At the stage of interiorization has been able to read matters relating to the given problem but have not been able to identify previous activities with the given problem, for coordination has not been able to coordinate the results of interiorization but was able to transform information into a mathematical model, the process of encapsulation has been able to build the problem structure and coordinate it but still not able to develop it, for generalization has not been able to, on the generalization process is still not able to use the existing scheme.

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