Student difficulties in solving covariational problems

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Abstract
This study discusses the difficulties of students in solving covariational problems. The study was conducted on 25 students with 6th-semester studies who had completed calculus courses. Students are asked to verbalize what they think while solving problems (think aloud). Next, students are interviewed with an in-depth interview technique to explore the thinking process. The results showed 88% of students had difficulties when given covariational problems. Students are familiar with mathematical problems with counting operations. When given a problem without information, students experience difficulties.

Keywords:
Covariational reasoning; students’ difficulties; covariation.

1 INTRODUCTION
Covariation became known in the late 1980s (Thompson, 1988) and early 1990 (Thompson, 1993a). According to Confrey, (Confrey, 1991; Confrey and Smith, 1994, 1995), covariations are patterns of quantity changes that can predict changes in changes in other quantity. Contrary to this opinion, Thompson (Thompson, 1994; Thompson, 1993b) states that covariation is the activity of coordinating two quantities that change simultaneously, which are not limited to values. After that, Carlson et al. (2002) constructed a framework for covariational reasoning, which is now the basis of research in subsequent covariational reasoning. Covariational reasoning is the essence of calculus. Calculus itself is the foundation that becomes the basic concept for understanding the next subject in the curriculum in Higher Education majoring in Mathematics Education. But the fact is teachers (Thompson et al., 2017), preservice teachers, students (Moore, 2014; Moore & Carlson, 2012; Paolelli and Moore, 2017); students (Ferrari-Escolá, Martínez-Sierra, and Méndez-Guevara, 2016); face difficulties in covariational reasoning. Therefore, strengthening the preparation of covariational reasoning competencies for prospective mathematics teachers needs special attention at the world level.

Covariational reasoning and the ability to understand graphics are an integral part of the calculus. Furthermore, the role of covariational reasoning and understanding graphics contribute significantly to progress in areas such as business, marketing (Arnould & Dion, 2018); medicine (Ito et al., 2017; Paulo & June, 2013); population mapping (Fountoulakis et al., 2008); chemistry (Nilov & Smolyakov, 2016); physics (Howell et al., 2009); police (Umoquit et al., 2008; Verhoff, 2008); Geology (Faggella et al., 2018); and much more. The importance of these benefits is shown through the many efforts that have been made to improve covariational reasoning competencies in understanding graphs (Carlson, 1998; Carlson et al., 2002; Madison, Carlson, Oehrtman, & Tallman, 2015; Moore & Carlson, 2012; Moore, Silverman, et al., 2013; Paolelli & Moore, 2017).

Carlson (1998) reveals the difficulties of students in connecting information that has been known to solve problems that are not common. Furthermore, Carlson et al. (2002) revealed that students experience difficulties in constructing graphs that change continuously and are not accurate in interpreting graph curvature in actual events. Moore et al. (2013) described the constructs of students’ thinking when constructing charts through clinical interviews. The results of his research also show that students have difficulty in interpreting attributes on the graph (Moore, 2013; Moore et al., 2013; Thompson, 1994). Indonesia is one of the biggest countries with the largest population in the world and makes education as the main focus in the development of human resources. Therefore, strengthening covariational reasoning needs to be done to increase the understanding of prospective teachers. For this reason, efforts to find out the covariational reasoning of prospective teachers in constructing the graph in solving covariational problems need to be done (NCTM, 2000).

2 METHOD
This research is qualitative research. An explanation of the students' reasoning process in constructing the solution is explained descriptively (Creswell, 2013). The study was conducted on 25 students 6th semester majoring in Mathematics Education, Universitas Negeri Malang. The reason for choosing the student because they have completed calculus courses. Research data are in the form of student work, student recordings during problem-solving, and student recordings during in-depth interviews. Participants are asked to say what they think (think aloud) during the problem-solving process. The interview was conducted twice, namely: in the first interview, it was aimed at confirming that students had never been given such a problem, while the interview aimed to dig up information that was not obtained when students solved the problem and confirmed the uncertain information. The research subject was a participant with the wrong answer. This reason for choosing the student because they have completed calculus courses. The results of his research also show that students have difficulty in interpreting attributes on the graph (Moore, 2013; Moore et al., 2013; Thompson, 1994). Indonesia is one of the biggest countries with the largest population in the world and makes education as the main focus in the development of human resources. Therefore, strengthening covariational reasoning needs to be done to increase the understanding of prospective teachers. For this reason, efforts to find out the covariational reasoning of prospective teachers in constructing the graph in solving covariational problems need to be done (NCTM, 2000).
with the aim of the problem being able to explore students' covariational reasoning. Covariational problems are shown in figure 1.

The reason why researchers use the problem as in Figure 1 is to know the thinking of the subject when given a problem that does not contain the attributes of numbers, and the subject can represent what is understood in any form. In this study, subjects were asked to perform translation representation from the pictorial representation to graphical representation. After the data is collected, the data will be analyzed by Carlson mental action of the covariation framework (see Table 1). With the disclosure of student difficulties when solving the covariational problem expected, educators can design problems and learning that can improve the sensitivity of students in reading events from a graph of dynamic events.

### Table 1. Carlson mental action of the covariation framework

<table>
<thead>
<tr>
<th>Mental Action</th>
<th>Description of Mental Action</th>
<th>Behaviors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mental Action 1 (MA1)</td>
<td>Coordinating the values of one variable with changes in the other</td>
<td>Labeling the axes with verbal indications of coordinating the two variables (e.g., y changes with changes in x)</td>
</tr>
<tr>
<td>Mental Action 2 (MA2)</td>
<td>Coordinating the direction of change of one variable with changes in the other variable</td>
<td>Constructing an increasing straight line</td>
</tr>
<tr>
<td>Mental Action 3 (MA3)</td>
<td>Coordinating the amount of change of one variable with changes in the other variable</td>
<td>Verbalizing an awareness of the direction of change of the output while considering changes in the input.</td>
</tr>
<tr>
<td>Mental Action 4 (MA4)</td>
<td>Coordinating the average rate-of-change of the function with uniform increments of change in the input variable</td>
<td>Plotting points/constructing secant lines</td>
</tr>
<tr>
<td>Mental Action 5 (MA5)</td>
<td>Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function</td>
<td>Verbalizing an awareness of the amount of change of the output while considering changes in the input.</td>
</tr>
</tbody>
</table>

### Results

Research data shows that very few participants can solve the bottle problem correctly. For more detailed results regarding participant answers, can be seen in table 2.

<table>
<thead>
<tr>
<th>No</th>
<th>Type of Student's Answer</th>
<th>Amount</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Right Answer</td>
<td>3</td>
<td>12%</td>
</tr>
<tr>
<td>2</td>
<td>Wrong Answer</td>
<td>22</td>
<td>88%</td>
</tr>
</tbody>
</table>

Based on the data presented in Table 2, this shows that 88% of students face difficulties when faced with the bottle problem. The researchers took one of the sample participants' wrong answers for similar answers. So, researchers only present one representative of the participant's answer. In this study, researchers classified three similar answers from each of the mistakes made by participants. For a clearer explanation, shown below. For directionless errors, occur in subject 1 (S1). If viewed at a glance, S1 has good coordination in constructing the graph. However, if seen more seriously. S1 determines the variable x is the time and determines the variable y is the height of the fuel. S1 starts the point on the y-ordinate at the top point to the right until y = 0. Graph S1 describes the opposite condition of filling water in the bottle. To be clearer about the results of the S1 work presented in Figure 2.
To find out more clearly the reason S1 draws a graph like that. Furthermore, the researchers interviewed in depth the S1 to find out the thinking process of students when constructing the graph. The results of the interview are shown below.

Interviewer: Explain why you are describing a graph like this? (the graph was wrong)
S1: At first, I set the x and y values first.

Interviewer: So, what is the x and y value?
S1: The time is x, so the fuel is high.

Interviewer: How come there are terms of time and high fuel? Does the information exist in the question?
S1: (S1 reads the question again). Oh yeah, yes, it should be the volume of water. Then y is high in water.

Interviewer: Sure?
S1: Hmmm... Sure sir

Interviewer: Oh yeah, why is your graph like this?
S1: (S1 smiles).

Interviewer: where is your starting point to draw a graph?
S1: (S1 uses her hand to point at the peak point y when x = 0)

Interviewer: Why?
S1: Because the water is filled into the bottle, sir.

Interviewer: what does this curved graph mean?
S1: this is because the bottom of the bottle is small (area a), the middle of the big bottle (area b), then the part toward the neck of the bottle is small again (area c), then the neck of the bottle is straight, so it has the smallest area (area d)

Interviewer: Oh, so you construct a graph based on the volume of water?
S1: Yes, sir. I focus on the volume of water.

Based on the results of the interview above shows that S1 draws a graph based on the volume of water present in the bottle without regard to changes in the volume of water to the height of water. S1 does not coordinate at all. For the direction of the graph that moves from top to bottom shows S1 does not pay attention to where the graph is. S2 also experienced errors. S2 cannot coordinate, so he cannot determine which independent variable and dependent variable. S2 determines height as x, and volume as y value. This can be seen in the results of the S2 work presented in Figure 3.

To know in more detail about the process of why S2 constructs the graph, as shown in Figure 3. The researchers interviews the S2. The interview results are written below:

Interviewer: Explain why you are describing a graph like this? (the graph was wrong)
S2: At first, I set the x and y values first.

Interviewer: So, what is the x and y value?
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S2: The height is x, so the volume is y.
Interviewer: Why are the graph shapes that you construct like this?
S2: Because the water volume is small, so the water level is still low. Then in this area (the S2 points to the second graph), the volume is large, so when the water increases slowly. Then in this area (the S2 points to the third graph), the volume of water is a little high because the shape is small.
Interviewer: What is the meaning of a small bottle?
S2: The width of the bottle (the cross-sectional area of the bottle) affects the speed or the slowness of the water level.
Interviewer: Then, on what is the first graph? (Refers to the first designated chart)
S2: Same thing, the width of the bottle affects the speed of the water level; it's just that I didn't write it down.

Based on the results of the interview, S2 has reasoned with covariational reasoning. S2 has realized that the wider the cross-section of the bottle, the increase in water is slower, the narrower the cross-section of the bottle, the faster the increase in water. S3 also experienced an error. The answer from S3 approaches the correct answer. However, there has been a slight error. There are also S3 work results presented in Figure 4.

![Figure 4](image)

Figure 4. The work results of the S3 in constructing a graph of volume water to the fuel height

To know in more detail about the process of why S3 constructs the graph, as shown in Figure 4. The researchers interviews the S3. The interview results are written below:

Interviewer: Explain why you are describing a graph like this? (the graph was wrong)
S3: At first, I set the x and y values first.
Interviewer: So, what is the x and y value?
S3: The volume of water is x, so the water height is y.
Interviewer: Why are the graph shapes that you construct like this?
S3: Because the cross-sectional area of the bottle affects the speed of increasing the water level.
Interviewer: Based on the graph that you have constructed. You write a similar sign. What does it mean?
S3: Because the shape of the bottle in this area (the S3 hand points to the bottle at the bottom) has the same shape as the bottle in this area (the S3 hand points to the bottle at the top). So I stated that it was similar.

Based on the results of the interview, the S3 answer indicated that he had reasoned with covariational reasoning. However, S3 does not realize that the bottle problem is a covariational problem of dynamic events. For a deeper discussion of the mistakes made by the subject when constructing a graph of changes in volume to height can be read in the discussion section.

4 DISCUSSION

Based on research data, these data show that 88% of students have difficulties when faced with problems that require covariational reasoning. These results indicate that the results of this study have similar results with previous studies (Carlson, 1998; Carlson et al., 2002; Carlson, Larsen, & Jacobs, 2001; Madison et al., 2015; Moore & Carlson, 2012; Moore & Paolletti, 2013; Moore et al., 2019; Paolletti & Moore, 2017; Weber & Thompson, 2014; Wilkie, 2019) that problems that require covariational reasoning are still difficult for most students. Specifically, in Indonesia, mathematics is identical to the calculation operation. Therefore, mathematical information is a number that is used to solve problems. When students are faced with problems that only require concepts to solve, this becomes an unusual problem.

The bottle problem is a new thing for students. Therefore, when given the bottle problem students seemed confused. This information is obtained from the results of the first interview before students solve the problem. All students involved in this study stated that "I have never faced a problem like this." So, this is the first time for students to be given a problem like this. At S1, it appears that the subject is unable to do covariational reasoning. S1 only focuses on variable volume and ignores high water variables. This is not surprising, because at the beginning of the S1 activity determining the independent variable is time and the dependent variable is the height of the fuel. After interviewing, S1 revises the determination of which dependent variable and independent variable to construct the graph. S1 also cannot determine
which independent variable and bound to the bottle problem. This shows that S1 does not understand perfectly in understanding the occurrence of the bottle problem. One of the factors causing this type of error is the weak ability to read S1. Whereas, Carlson, Madison, and West (2015) stated that reading ability was the main factor for students to understand the occurrence of problems. Another contributing factor is that S1 is unable to coordinate the information provided (Carlson et al., 2002). Meanwhile, to solve problems like this, students must have a good concept.

S2 has done covariational reasoning in completing the bottle problem even though not maximally. We can say that S2 has a fairly good calculus concept. This can be seen when S2 describes the graph. Each S2 describes a graph, although it is not verbally acknowledged. S2 reflects on each process he describes the graph. In the first area, the S2 only focuses on the volume of water and does not see the cross-sectional area of the bottle as the cause of the increase in water level. This can be known through the reason written on the answer sheet "the volume of water is small, so the height is still low." In this statement, the students' thoughts are only focused on the volume of water. When the volume of water is low, the water level is also low. In the second area, the mindset has changed. S2 has noticed the time variable implicitly in the bottle problem. S2 states that "the volume of water is large, the time of change in water level is long." From the statement, S2 began to pay attention to time as one of the influencing variables. Although, in constructing a graph, the time variable does not exist explicitly. However, implicitly also becomes one of the factors for constructing the graph. In the third area, reflection is experienced again by the S2. S2 considers the shape of the bottle, which affects the speed of the water level. This was indicated by the S2 through a written statement "the volume of water is small when the water rises so fast. Because the shape is small". When confirmed through interviews, S2 stated that "the shape of the bottle affects the speed of the water level." S2 has carried out Mental Action 5 on the framework of covariational reasoning but, pseudo.

This can be seen through the S2's actions in making mistakes in determining the independent variables and dependent variables in constructing the graph. S2 determines the height of the water as an independent variable and the volume of water as a dependent variable. When the beginning of the S2 was correct in describing the graph. However, S2 was not sure of the graph he had constructed. This can be seen through the recordings that show that S2 revised the graphic form four times. Between S1, S2, and S3. S3 has the best covariational reasoning skills. S3 has been able to determine that the volume of water is an independent variable (x-axis) and the height of the water is a dependent variable (y-ordinate). Since the beginning, S3 has outlined what information is the determining factors for constructing the graph. S3 has underlined the shape of the bottle affecting the speed of increasing water levels. This was shown through S3 interview results which stated that "Because the cross-sectional area of the bottle is increasing the water level." S3 constructs a graph by dividing it into four parts; this information is obtained through a graph form that he has constructed. At the bottom of the bottle, S3 constructs a graph with a steep slope and indicates that the cross-sectional area of the bottle in that section is narrow. In the middle part of the bottle, S3 constructs a graph with a gentle slope and indicates that the cross-section of the bottle is broad. At the top of the bottle, S3 creates a graph with a steep slope and is the same as the first graph. At the top of the bottle (neck), S3 constructs a graph with a sloping slope again.

Unfortunately, S3 doesn't see the shape of a bottle that curves as a dynamic event. In the first and third parts of the marked graph S3 with the "similar" statement, S3 only looks globally and does not specifically view the event. The occurrence of the bottle part in the first part, with the third bottle part having the same thing. However, the incident must be reversed. S3 is not aware of this. Based on the discussion that has been described, the strengthening of covariational reasoning competencies needs to be of particular concern to prospective teachers. Furthermore, especially for educators, covariational reasoning also needs to be prepared since students are in the position of primary school especially in Indonesia given that several other countries have provided efforts to strengthen covariational reasoning from an early age (Ferrari-Escolá et al., 2016; Yemen-Karpuzcu et al, 2015). This study certainly has weaknesses; this study is limited to a small number of subjects. With a greater number of subjects and at a variety of levels, it allows the emergence of variations in student completion and covariational reasoning processes.

5 CONCLUSION

In Indonesia, covariational reasoning is still an unfamiliar thing. Students in Indonesia are used to solving mathematical problems with counting operations. When faced with a problem that as a whole involves concepts and reasoning, students experience difficulties. Therefore, this needs to be of particular concern in the field of Indonesian Education, especially Mathematics Education.

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